A NEW SPINNING MEMBRANE LAGRANGIAN submitted to the Journal of Group Theory in Physics

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(January 1990; Submitted March 1993)

Abstract

A new local world volume supersymmetric Lagrangian for the bosonic membrane is presented. The starting Lagrangian is the one constructed by Dolan and Tchrakian with vanishing cosmological constant, with quadratic and quartic derivative terms. Our Lagrangian differs from the one constructed by Lindstrom and Rocek in the fact that it is polynomial in the fields facilitating the quantization process. It is argued, rigorously, that if one wishes to construct polynomial actions without a curvature scalar term and, where supersymmetry is linearly realized in the space of physical fields, after the elimination of auxiliary fields, one must relinquish S supersymmetry, altogether, and concentrate solely on the Q supersymmetry associated with the superconformal algebra in three dimensions. A full $''Q+S''$ supersymmetry cannot be implemented in a linearly realized way satisfying all of the above-mentioned requirements, unless a non-polynomial action is chosen.

PACS:04.65.+e, 04.20.Fy.

1 Introduction

In the past years there has been considerable progress in the theory of two dimensional extended objects; i.e. membranes. However, a satisfactory spinning membrane Lagrangian has not been constructed yet, as far as we know. Satisfactory in the sense that a suitable action must be one which is polynomial in the fields, without R terms (curvature scalar) which interfere with the algebraic elimination of the three-metric, and also where supersymmetry is linearly realized in the space of physical fields. Howe, Duff and others [1] [2] sometime ago mentioned the fact that it is allegedly impossible to supersymmetrize Dirac-Nambu-Goto type of actions (DNG) -those proportional to the world-sheet and world-volume spanned by the string (membrane) in their motion through an embedding space-time. The efforts to supersymmetrize this action have generally been based upon the use of the standard, classically-equivalent, bosonic action which included a cosmological constant. The supposed obstruction is related to the fact that in order to supersymmetrize this constant one had to include an Einstein-Hilbert term spoiling the process altogether.

Bergshoeff et al [3] went evenfurther and presented us with the "no-go" theorem for the spinning membrane. Their finding was based in the study of a family of actions, in addition to the one comprised of the cosmological constant, which were equivalent, at the classical level,to the DNG action. However, this "no-go" theorem was flawed because these authors relied on the tensor calculus for Poincare D=3 N=1 SG developed by Uematsu. [4] Unfortunately, the above tensor calculus does not even yield a linearly realized supersymmetry for the kinetic matter multiplet to start with!. A constraint, $\bar{\chi}\chi = 0$, appears after the elimination of the S auxiliary field where χ is the three dimensional Majorana spinor. By the spinning membrane one means that one has supersymmetry on the world-volume whereas by the supermembrane one means that one has supersymmetry on the embedding spacetime background. Lindstrom and Rocek [5] were the first ones to construct a Weyl invariant spinning membrane action. However, such action was highly non-polynomial complicating the quantization process evenfurther than the one for the supermembrane.

The suitable action to supersymmetrize is the one of Dolan and Tchrakian [6] without a cosmological constant and with quadratic and quartic-derivative terms. A class of conformally-invariant σ- model actions was shown to be equivalent, at the classical level, to the DNG action for a $p+1$ extended object ($p+1$=even) embedded in a target spacetime of dimension $d \geq p+1$. When p+1=odd, our case, an equivalent action was also constructed, however, conformal invariance was lost in this case. The crux of this work lies on the necessity to embed the Dolan-Tchrakian action in a Weyl-invariant one through the introduction of extra fields. These are the gauge fields of dilations, b_μ, and the scalar coupling of dimension $(length)^{+3}$, A_0, which appears in front of the quartic derivative terms. The latter must appear with a suitable coupling constant in order to render the action dimensionless. Upon embedding the Dolan-Tchrakian action in a Weyl invariant one this coupling constant becomes a scalar. A similar procedure occurs in the Brans-Dicke formulation of gravity.

Having gone through the embedding procedure the natural question to ask is how do we eliminate these new fields, b_μ, A_o, in order to recover the original action? One will recover back the Dolan-Tcrakian action by fixing the dilational invariance :$A_0 = constant$ and enforcing $b_\mu = 0$. This is achieved, simultaneously, if one imposes the natural constraint on A_0: $D_\mu^{Weyl} A_0 = \partial_\mu A_o + 3 b_\mu A_o = 0$. Such constraint can vbe derived from first principles: from an action. It follows automatically that if the equations of motion of the Weyl-covariantized Dolan-Tchrakian action are, indeed,the Weyl-covariant extension of the Dolan-Tchrakian equations of motion, then $D_\mu A_o = 0$ follows immediately. The reason why we cannot fix the conformal boost gauge invariance by setting :$b_\mu = 0$, is because our final spinning membrane action is not invariant under conformal boosts (the b_μ field does not decouple) nor under S supersymmetry. Therefore, on-shell dilational gauge invariance of the Weyl covariantized Dolan Tchrakian (WCDT) action allows us to recover the original DT action. Notice that we have not imposed any constraints, whatsoever, on our physical fields!. (See Appendix for further details).

Once the embedding process is performed one supersymmetrizes the WCDT action by incorporating A_o into a superconformal coupling-function multiplet (A_o, χ_o, F_o). The field b_μ is part of the superconformal gauge multiplet involving $(e_\mu^m, \psi_\mu, b_\mu)$, and the physical fields of the membrane form part of the superconformal matter multiplet, (A, χ, F). The A^i fields are identified with the membrane's embedding coordinates, X^i. Finally, if we wish to eliminate any curvature scalar terms one must take suitable combinations of products involving these three superconformal multiplets and, in doing so, one is going to break, explicitly, the S supersymmetry of the three-dim superconformal algebra as well as the conformal boost symmetry, the K symmetry, which signals the presence of the b_μ field in our final spinning membrane action.

The final action is Lorentz, dilational, Q supersymmetric and translational invariant. There is nothing wrong with these fact since the subalgebra of the full three-dim superconformal algebra comprised of the Lorentz generator, dilations, Q supersymmetry and translations, P_μ, does indeed close !! In conventional Poincare supergravity one has an invariance under a particular linear combination of Q and S supersymmetry : the so-called $Q+S$ rule as well as K symmetry which enforces the decoupling of the b_μ field from the supersymmetric action. Here we have a different picture, we have full Q supersymmetry, instead of a particular combination of Q and S, and no conformal boost invariance. This is the main peculiarity of our spinning membrane action and the most important result from the group theoretical point of view.

The outline of the paper goes as follows:

In section II we discuss the work of Dolan and Tchrakian (DT) and show the equivalence to DNG type of actions. In section III we present the problems associated with the Poincare tensor calculus and point out why the "no-go" theorem was inappropriate. In the final section, IV, we give a detail argument showing that in order to satisfy all of the stringent requirements discussed earlier, we must relinquish S-supersymmetry and concentrate, solely, on the Q-supersymmetry associated with the superconformal algebra in three dimensions. "$Q+S$" supersymmetry can only be implemented in the class of non-polynomial actions (if we insist in meeting all of our requirements) as it was shown by Lindstrom and Rocek . Finally, the fully Q-invariant action is presented in 4.1; In subsection 4.2 we discuss in full detail how to retrieve the original Dolan-Tchrakian action, after eliminating the auxiliary fields and setting the fermions to zero, and why, then, we have a Q spinning membrane. An appendix is included where we derive from first principles the embedding condition :$D_\mu A_o = 0$, which enables us to set $b_\mu = 0$, while fixing A_o to a constant recovering in the process the original DT action.

Our coventions are: Greek indices, $\mu, \nu...$ stand for three-dimensional world volume ones: $0, 1, 2$; Latin indices, $i, j, k...$ for spacetime ones.

2 The Dolan-Tchrakian Action

The new Lagrangian for the bosonic m-extendon (m-brane) with vanishing cosmological constant constructed by Dolan and Tchrakian for $m = odd$. $m+1 = 2n$, is :

$$L_{2n} = \sqrt{-g} g^{\mu_1\nu_1} .... g^{\mu_n\nu_n} \partial_{[\mu_1} X^{i_1} ...... \partial_{\mu_n]} X^{i_n} \partial_{[\nu_1} X^{j_1} ...... \partial_{\nu_n]} X^{j_n} \eta_{i_1 j_1} .... \eta_{i_n j_n}. \quad (2-1)$$

η_{ij} is the spacetime metric and $g^{\mu\nu}$ is the metric for the $2n$ dimensional hypersurface spanned by the m-extendon and the antisymmetrization of indices is explicitly shown. Upon elimination of the $2n \times 2n$ matrix :

$$A^\mu_\nu = g^{\mu\rho} \partial_\rho X^i(\sigma) \partial_\nu X^j(\sigma) \eta_{ij}. \quad (2-2)$$

from the n^{th} order polynomial in A :

$$A^n - b_{2n-1} A^{n-1} + b_{2n-2} A^{n-2} - ....(-1)^{n+1} b_{n+1} A + 1/2 \ (-1)^n b_n I_{2n\times 2n} = 0. \quad (2-3)$$

where the scalar coeffifcients b^i are the same as the first $n+1$ coefficients in the expansion :

$$det(A - \lambda I) = \lambda^{2n} - b_{2n-1} \lambda^{2n-1} + b_{2n-2} \lambda^{2n-2} .... - b_1 \lambda + detA. \quad (2-4)$$

and substituting the solution of (2-3) back into (2-1) one finds :

$$L_{2n} = \sqrt{-det(\partial_\mu X^i \partial_\nu X^j \eta_{ij})} [(n!)^2 b_n / \sqrt{A}]. \quad (2-5)$$

The crucial observation made by Dolan and Tchrakian was that the last factor in (2-5) takes discrete values for all n. Therefore, the equivalence to the Nambu-Goto action. Notice that for every n, L_{2n} is conformally invariant under scalings and despite L_{2n} being of order $2n$ in derivatives it is only quadratic in time derivatives because of the antisymmetry in the wedge type of product in (2-1). Therefore, attempts to quantization might not be hopeless.

When $m = even; m+1 = 2n+1$ a Lagrangian with zero cosmological constant can also be constructed; however, conformal invariance is lost. For our case, the membrane. $m = 2$ the Lagrangian is :

$$L_{mem} = L_4 + aL_2. \tag{2-6}$$

$$L_4 = \sqrt{-g}g^{\mu\nu}g^{\rho\tau}\partial_{[\mu}X^i\partial_{\rho]}X^k\partial_{[\nu}X^j\partial_{\tau]}X^l\eta_{ij}\eta_{kl}. \tag{2-7}$$

and

$$L_2 = \sqrt{-g}g^{\mu\nu}\partial_\mu X^i\partial_\nu X^j\eta_{ij}. \tag{2-8}$$

where $\mu,\nu = 0,1,2; a = constant; i,j... = 1,2,....d$. The above Lagrangian upon elimination of the world volume metric is :

$$12\sqrt{a}\ \sqrt{-detG}\ or\ -4\sqrt{a}\ \sqrt{-detG}. \tag{2-9}$$

with $G_{\mu\nu} = \partial_\mu X^i\partial_\nu X^j\eta_{ij}$.

3 A nonlinearly realized supersymmetric membrane

We shall begin by writing down the supersymmetric term (modulo total derivatives) for the Poincare kinetic scalar multiplet in D=3 N=1 SG given by Uematsu :

$$L = \frac{1}{2}[\Sigma_P \otimes T(\Sigma_P)]_{inv} - \frac{1}{4}[T(\Sigma_P \otimes \Sigma_P)]_{inv} =$$
$$e[-1/2\ g^{\mu\nu}\partial_\mu A\partial_\nu A - 1/2\ \bar{\chi}\gamma^\mu D_\mu\chi + 1/2\ F'^2 + 1/2\ \bar{\psi}_\nu\gamma^\mu\gamma^\nu\chi\partial_\mu A +$$
$$1/16\ \bar{\chi}\chi\bar{\psi}_\nu\gamma^\mu\gamma^\nu\psi_\mu + 1/8\ S\bar{\chi}\chi]. \tag{3-1}$$

The kinetic multiplet -which is not uniquely defined since it is defined up to the addition of a fixed scalar multiplet which starts with SA as its first component- is given by:

$$\tilde{A} = F'.\quad \tilde{\chi} = \gamma^\mu D^P_\mu\chi - 1/4\ S\chi. \tag{3-2a,b}$$

$$\tilde{F}' = e^{-1}\partial_\mu(eg^{\mu\nu}D^P_\nu A) - 1/2\ \bar{\psi}^\nu\gamma^\mu\psi_\mu D^P_\nu A - 1/2\ \bar{\psi}^\mu D^P_\mu\chi +$$
$$i/4\ e^{-1}\epsilon^{\mu\nu\rho}\bar{\chi}\gamma_\rho\psi_{\mu\nu} - F'S + 1/8\ S\bar{\psi}_\mu\gamma^\mu\chi. \tag{3-2c}$$

Notice that (3-1) is ,by itself, unsatisfactory because upon elimination of S via its equations of motion one ends up with an unnatural constraint among the physical fields of the theory ,$\bar{\chi}\chi = 0$. Similar results are obtained when we supersymmetrize the quartic terms. The appearace of such constraint after the elimination of S traces all the way back to the tensor calculus and transformation rules for Poincare D=3 N=1 SG given by Uematsu. Such rules are essentially identical to the two-dim case except for a minor modification in the transformation law of the matter auxiliary field, F'. This, in turn, forces an extra term, linear in S, in the $\tilde{\chi}; \tilde{F}'$ components of the kinetic multiplet. Armed with these minor changes, one obtains (3-1). These modifications "propagate" to the quartic terms also and unwanted linear couplings among S and the matter fields appear forcing unnatural constraints after the elimination of S. In order to remedy this one could add the pure

Supergravity cation with a corresponding S^2 term. However, this is precisely what we wanted to avoid : the presence of R terms in our action !.

There are ways to circumvent this problem. One way was achieved by Linstrom and Rocek who started from a non-polynomial but conformally invariant action :

$$I \sim \int d^3\sigma \sqrt{-g}(g^{mn}\partial_m X^\mu \partial_n X^\nu \eta_{\mu\nu})^{3/2}. \qquad (3-3)$$

where $m,n = 0,1,2.\quad \mu,\nu = 1,2.......d$. Since S is an "alien" concept in conformal supergravity, it cannot appear in the supersymmetrization process, whether one uses Conformal or Poincare supergravity techniques to build invariant actions. This was explictly verified by Lindstrom and Rocek. The shortcoming is that the action is highly non-polynomial complicating evenfurther the quantization process than the one of the supermembrane. On the other hand, the Dolan-Tchrakian action is polynomial but as a result of the non-linearly realized supersymmetry due to the matter fields constraints (upon elimination of S) the quantization programme is going to be hampered considerably. In the next section we present ways to supersymmetrize the Dolan-Tchrakian action. The crucial difference is that we shall only implement Q supersymmetry instead of both Q and S supersymmetry of the superconformal algebra in three dimensions.

4 The Lagrangian

4.1 A Q Spinning Membrane

In this section we will present an action for the 3-dim- Kinetic matter supermultiplet where supersymmetry is linearly realized and without R terms. Also we will supersymmetrize the quartic-derivative terms of (1-1). This is attained by using directly an explicitly superconformally invariant action for the kinetic terms. The quartic terms do not admit a superconformally invariant extension as we shall see shortly. The key issue lies in the fact that if we wish to satisfy the three requirements:

1). A spinning membrane action which is polynomial in the fields.

2) Absence of R terms.

3). Linearly realized supersymmetry in the space of fields after the elimination of the auxiliary fields, before and after one sets the Fermi fields to zero.

One must relinquish S-supersymmetry altogether and concentrate solely on the Q-supersymmetry associated with the superconformal algebra in D=3. We shall begin with some definitions of simple-conformal SG in D=3. [4]:

The scalar and kinetic multiplet of simple conformal SG in D=3 are respectively:

$$\Sigma_c = (A,\chi,F).$$

$$T_c(\Sigma_c) = (F, \not{D}^c\chi, \Box^c A) \qquad (4-1)$$

We have the following quantities:

$$D^c_\mu A = \partial_\mu A - 1/2\bar{\psi}\chi - \lambda b_\mu A. \qquad (4-2)$$

$$D^c_\mu\chi = (D_\mu - (\lambda+1/2)b_\mu)\chi - 1/2\ \not{D}^c A\psi_\mu - 1/2F\psi_\mu - \lambda A\phi_\mu. \qquad (4-3)$$

$$\Box^c A = D^c_a D^{ca} A = e^{-1}\partial_\nu(eg^{\mu\nu}D^c_\mu A) + 1/2\bar{\phi}_\mu\gamma^\mu\chi - (\lambda-1)b^\mu D^c_\mu A +$$

$$2\lambda A f^a_\mu e^\mu_a - 1/2\bar{\psi}^\mu D^c_\mu \chi - 1/2\bar{\psi}^\mu \gamma^\nu \psi_\nu D^c_\mu A. \tag{4-4}$$

$$\omega^{mn}_\mu = -\omega^{mn}_\mu(e) - \kappa^{mn}_\mu(\psi) + e^n_\mu b^m - e^m_\mu b^n. \tag{4-5-a}$$

$$\phi_\mu = 1/4\gamma^\lambda\gamma^\sigma\gamma_\mu S_{\sigma\lambda} = 1/2\sigma^{\lambda\sigma}\gamma_\mu S_{\sigma\lambda}. \tag{4-5-b}$$

$$\kappa^{mn}_\mu = 1/4(\bar{\psi}_\mu\gamma^m\psi^n - \bar{\psi}_\mu\gamma^n\psi^m + \bar{\psi}^m\gamma_\mu\psi^n).$$

$$S_{\mu\nu} = (D_\nu + 1/2\ b_\nu)\psi_\mu - \mu \leftrightarrow \nu. \tag{4-5-d}$$

$$e^{a\mu} f_{a\mu} = -1/8R(e,\omega) - 1/4\bar{\psi}_\mu\sigma^{\mu\nu}\phi_\nu. \tag{4-5-e}$$

The transformation laws under Weyl scalings, Q and S supersymmetry are respectively:

$$\delta e^m_\mu = \lambda e^m_\mu;\quad \delta g^{\mu\nu} = 2\lambda g^{\mu\nu};\quad \delta A = 1/2\lambda A;\quad \delta\chi = \lambda\chi; \delta F = 3/2\lambda F \tag{4-6}$$

$$\delta^c_Q e^m_\mu = \bar{\epsilon}\gamma^m\psi_\mu;\quad \delta^c_Q\psi_\mu = 2(D_\mu + 1/2b_\mu)\epsilon;\quad \delta^c_Q b_\mu = \phi_\mu \tag{4-7a}$$

$$\delta^c_Q A = \bar{\epsilon}\chi;\quad \delta^c_Q\chi = F\epsilon + \not{D}^c A\epsilon.\quad \delta^c_Q F = \bar{\epsilon}\ \not{D}^c\chi. \tag{4-7b}$$

$$\delta^c_S b_\mu = \frac{-1}{2}\psi_\mu\epsilon_s. \tag{4-8a}$$

$$\delta^c_S\omega^{mn}_\mu = -\epsilon_s\sigma^{mn}\psi_\mu. \tag{4-8b}$$

$$\delta^c_S e^m_\mu = 0;\quad \delta^c_S\psi_\mu = -\gamma_\mu\epsilon_s \tag{4-8c}$$

$$\delta^c_S A = 0;\quad \delta^c_S\chi = \lambda A\epsilon_s;\quad \delta^c_S F = (1/2-\lambda)\bar{\chi}\epsilon_s. \tag{4-8d}$$

Notice that the kinetic multiplet transforms propely under Q-transformations for any value of the conformal weight, λ, but not under S transformations unless one assigns the canonical weight $\lambda_c = \frac{1}{2}$. Furthermore, if we wish to write down superconformally invariant actions [4] for a kinetic multiplet, we must have for Lagrangian :

$$L = e[F + \frac{1}{2}\bar{\psi}_\mu\gamma^\mu\chi + \frac{1}{2}A\bar{\psi}_\mu\sigma^{\mu\nu}\psi_\nu]. \tag{4-9}$$

and make sure to have built the kinetic multiplet from a matter multiplet whose $\lambda = \frac{1}{2}$ otherwise we would not even have Q-invariance despite the fact that the kinetic multiplet transforms properly under Q-transformations irrespectively of the value of λ. On physical grounds we see that the notion of canonical dimension is intrinsically tied up with the conformal invariant aspect of the kinetic terms in the action. We have a conformally invariant kinetic term if,and only if, the fields have the right (canonical)dimensions to yield terms of dimension three in the Lagrangian. We might ask ourselves how did Lindstrom & Rocek manage to construct a Weyl invariant spinning membrane when their fields had a non-canonical dimension? The answer to this question lies on the nonpolynomial character of their action. Formally one has an infinite series expansion where each explicitly Q and S-breaking term is cancelled by the next term in the expansion. The task now is to see how do we write a suitable action for the kinetic terms without R terms (which appear in the definition of the D'Alambertian) for values of λ different than zero. The suitable action is obtained as follows:

Take the combination $\Sigma_C^i \otimes T_C(\Sigma_C^j) + T_C(\Sigma_C^i) \otimes \Sigma_C^j - T_C(\Sigma_C^i \otimes \Sigma_C^j)$ which happens to be the correct one to dispense of the R terms. The explicit components of the latter multiplet are (Notice the $b\mu$ terms):

$$A = \bar{\chi}_i \chi_j. \qquad (4-10)$$

$$\chi = F_i \chi_j + F_j \chi_i - \frac{1}{2} \bar{\chi}_i \chi_j \gamma^\mu \psi_\mu - \not{\partial} A_i \chi_j - \not{\partial} A_j \chi_i. \qquad (4-11)$$

$$F = -2g^{\mu\nu} \partial A_i \partial A_j - 2\bar{\chi}_i \not{D} \chi_j + F_i F_j + \frac{1}{2} \bar{\chi}_i \gamma^\mu \gamma^\nu \psi_\mu \partial_\nu A_j$$
$$+(i \leftrightarrow j) + \bar{\chi}_i \psi^\mu \partial_\mu A_j + (i \leftrightarrow j) - \frac{1}{2} \bar{\chi}_i \chi_j \bar{\psi}_\nu \gamma^\mu \gamma^\nu \psi_\mu + \frac{1}{4} \bar{\chi}_i \chi_j \bar{\psi}^\mu \psi_\mu$$
$$+\frac{1}{2} \bar{\chi}^i \gamma^\mu \psi_\mu F^j + (i \leftrightarrow j) + \frac{1}{2} \bar{\chi}^i \gamma^\mu \phi_\mu A^j + (i \leftrightarrow j)$$
$$+\lambda e^{-1} \partial_\nu (e g^{\mu\nu} b_\mu A_i A_j) + \lambda b^\mu \partial_\mu (A_i A_j) - 2\lambda^2 b^\mu b_\mu A_i A_j. \qquad (4-12)$$

Unfortunately matters are not that simple! It is true that the components of the latter mutiplet transform properly under Q-transformations since each single one of the conformal Kinetic-multiplets in the definition of (4-10;11;12) does. However, this not the case for S-supersymmetry since the component, $T(\Sigma \otimes \Sigma)$, is not invariant under S-supersymmetry because the weight of $\Sigma \otimes \Sigma$ is equal to 1 instead of $\frac{1}{2}$. Therefore, eliminating the R terms is not compatible with S-supersymmetry. Of course, recuring to a non-polynomial action [5] allows for this possibility to occur since each term in the infinite series expansion compensates for the lack of S-supersymmetry of the previous one as we have already stated earlier. We are forced, then, to relinquish S-supersymmetry and implement Q-supersymmetry only.

Our action is Q-supercovariant and is obtained by plugging-in directly A, χ and F in equation (4-9) and contracting the spacetime indices with η_{ij} . It has a similar form as (3-1) but it does not contain the term linear in S, $S\bar{\chi}\chi$, exclusively ,which was the one which furnished the constraint between our physical fields in (3-1) after elimination of S. Furthermore, it contains the term $\frac{1}{2}\bar{\chi}^i \gamma^\mu \phi_\mu A^j$ which does not appear in (3-1). i.e; after a total derivative is performed we end up with $\frac{1}{2} A \bar{\phi}_\mu \gamma^\mu \gamma^\nu \partial_\nu \chi$. Moreover, we don't have R terms, Q-supersymmetry is linearly realized after the elimination of F or, if we wish, after eliminating F' and S once we set $F = F' + \frac{1}{4} AS$.

The derivatives in (4-10;11;12) contain the spin-connection which is a function of $e_\mu^a; \psi_\mu$ and b_μ. We could have presented the following argument in five steps that would have allowed us us to fix the conformal-boost invariance and set $b_\mu = 0$. This occurs if, and only if, our action is invariant under conformal boosts.(Unfortunately it is not so; however for the sake of completeness we shall go ahead).

1).The Q-superconformally invariant action comprised of (4-9) after plugging-in the values for $A; \chi$ and F given by (4-10;11;12) does not contain explicitly $f_{a\mu}$ given by (4-5-e)(The R terms cancel out as well as the subsequent Rarita-Schwinger terms).

2). The fields $e_\mu^a; \psi_\mu$ and the matter multiplet, Σ_C, are inert under K transformations (conformal boosts).[4].

3).Therefore, the variation of the Lagrangian with respect K-transformations is:

$$[\frac{\partial L}{\partial b_\mu}+\frac{\partial L}{\partial \omega}\frac{\partial \omega}{\partial b_\mu}+\frac{\partial L}{\partial \phi_\mu}\frac{\partial \phi_\mu}{\partial b_\mu}]\delta_K b_\mu+\frac{\partial L}{\partial f_{a\mu}}\delta_K f_{a\mu}=0. \qquad (4-13)$$

4). Therefore, b_μ decouples from the action if it is invariant under conformal boosts .

5). If b_μ decouples and , if our action was indeed invariant under Weyl scalings, this must be a signal that there is no need to use conformally covariant derivatives with respect to dilatations since the action was already Weyl invariant to begin with. Having followed the above five-step argument we can infer that for those actions which are K-invariant we can fix the conformal-boost invariance and set $b_\mu=0$.

This is all fine but is our action (the one given by eqs. 4-9;4-10;4-11;4-12) K-invariant? The answer is no. The b_μ terms do not decouple. We will relegate the discussion of these terms for the Appendix. Since the presence of these terms is harmless for the rest of the forthcoming discussion and results we will postpone, for the time being, the discussion of the b_μ terms until the Appendix.

Therefore, we have constructed an action which could not have been obtained by direct Poincare tensor calculus methods. i.e; invariant under Q but not S-transformations. This was the main reason why the "no-go" theorem was not quite correct : S-supersymmetry cannot be implemented in a linearly-realized way in the absence of R terms and the Poincare tensor calculus does not even yield a linearly-realized supersymmetry for the Kinetic terms to begin with!. An example of a multiplet which transforms properly under the $''Q+S''$ sum rule but not separately under Q nor S supersymmetry is the particular Poincare-Kinetic multiplet.[4]:

$$T_p(\Sigma_p)=(F;\, \not{D}^c\chi(\lambda=\frac{1}{2});\Box^c A-\frac{3}{4}FS).$$

This multiplet is "almost" identical to the superconformal one were it not for the $-\frac{3}{4}FS$ term. The last component of a Poincare multiplet is $F'=F-\frac{1}{2}\lambda AS$, where in the case above we have $\lambda=1+\frac{1}{2}$ for $T_p(\Sigma_p)$. Therefore, one can see that it transforms properly under the "Q+S" rule but not separately under both Q and S transformations.

Now we turn to the supersymmetrization of the quartic terms. Why do we need to do this if, perhaps, we could bypass it by working directly with the action containing the cosmological constant? What happens is that we don't retrieve the cosmological constant after eliminating S from an action comprised of (4-9), where we use for kinetic terms solely the piece, $\Sigma\otimes T(\Sigma)$, and the one constructed from the "constant" Poincare supermultiplet: $\Sigma_p=(1,0,0)$. i.e; one gets negative powers of the A field.

We proceed now to supersymmetrize the L_4 terms. One cannot obtain a superconformally invariant action (not even Q-invariant) now because these terms do not have the net conformal weight of $\lambda=2$ as the kinetic terms had. (We refer to the weight of the first component of a multiplet so that F has dimension three). For this reason we have to introduce the following coupling function, a multiplet, that has no dynamical degrees of freedom but which serves the purpose of rendering the quartic-derivative terms with an overall dimension three to ensure that our action is in fact dimensionless. We refrained from doing this sort of "trick" in the case of the kinetic terms because such terms are devoid of a dimensional coupling constant. The Dolan and Tchrakian's action contains an arbitrary constant in front of the quartic pieces and it is only the ratio between this constant and the dimensionless constant in front of L_2 which is relevant. This constant must have dimensions of $(length)^{+3}$ since we have an extra piece of dimension three stemming from the term ,$(\partial_\mu A)^2$.

Let us, then, introduce the coupling-function-multiplet

$$\Sigma_0=(A_0;\chi_0;F_0)$$

whose Weyl weight is equal to -3 so that the tensor product of Σ_0 with the following multiplet, to be defined below, has a net conformal weight , $\lambda = 2$ as it is required in order to have Q-invariant actions.

Lets introduce the following multiplet

$$K^{ijkl}_{\mu\nu\rho\tau}\eta_{ij}\eta_{kl} = K(\Sigma^i_\mu;\Sigma^j_\nu)\otimes T[K(\Sigma^k_\rho;\Sigma^l_\tau)] + (ij \leftrightarrow kl) and (\mu\nu \leftrightarrow \rho\tau) -$$

$$T[K(\Sigma^i_\mu;\Sigma^j_\nu)\otimes K(\Sigma^k_\rho;\Sigma^l_\tau)]\eta_{ij}\eta_{kl}$$

where the definition of $K(\Sigma,\Sigma)$ is:

$$K(\Sigma,\Sigma) = \Sigma^i_C \otimes T_C(\Sigma^j_C) + T_C(\Sigma^i_C)\otimes\Sigma^j_C - T_C(\Sigma^i_C\otimes\Sigma^j_C).$$

This multiplet is the adequate one to retrieve (2-7) at the bosonic level and also the one which ensures that the R terms do cancel from the final answer. This is indeed the case as it was shown in eqs- (4-10;11;12). After a tedious calculation we obtain the components of the supersymmetic-quartic-derivative terms: (Again, notice the b_μ terms which must be present because we have no longer conformal-boost invariance):

$$A = \bar{\chi^{ij}}\chi^{kl}. \qquad (4-14)$$

$$\chi = -\not\partial(\bar{\chi^i}\chi^j)\chi^{kl} + (i,j \leftrightarrow k,l) - \frac{1}{2}\bar{\chi^{ij}}\chi^{kl}\gamma^\mu\psi_\mu + (i,j\leftrightarrow k,l)$$
$$+\chi^{ij}F^{kl} + (i,j\leftrightarrow k,l). \qquad (4-15)$$

$$F = \bar{\psi^\mu}\partial_\mu(\bar{\chi^i}\chi^j)\chi^{kl} + \frac{1}{4}\bar{\psi^\mu}\psi_\mu\bar{\chi^{ij}}\chi^{kl}$$
$$+\bar{\chi^{ij}}[-2\not D\chi^{kl} + \frac{1}{2}\gamma^\nu\gamma^\mu\psi_\mu\partial_\nu(\bar{\chi^k}\chi^l) - \frac{1}{4}\chi^{kl}\bar{\psi_\mu}\gamma^\mu\gamma^\nu\psi_\nu + \frac{1}{2}\gamma^\mu\psi_\mu F^{kl}] +$$
$$F^{ij}F^{kl} - 2g^{\mu\nu}\partial_\mu(\bar{\chi^i}\chi^j)\partial_\nu(\bar{\chi^k}\chi^l) +$$
$$\frac{1}{2}\bar{\chi^{ij}}\gamma^\mu\phi_\mu\bar{\chi^k}\chi^l + (ij\leftrightarrow kl)$$
$$+\lambda e^{-1}\partial_\nu(eg^{\mu\nu}b_\mu\bar{\chi}_i\chi_j\bar{\chi}_k\chi_l) + \lambda b^\mu\partial_\mu(\bar{\chi}_i\chi_j\bar{\chi}_k\chi_l) - 2\lambda^2 b_\mu b^\mu\bar{\chi}_i\chi_j\bar{\chi}_k\chi_l. \qquad (4-16)$$

Where we have used the abbreviations χ^{ij} and F^{ij} already given in (4-10;11;12).

Notice the similarity between (4-14;15;16), above, and (4-10;11;12) in form and in the values of the coefficients.This is a sign of consistency. We need to take the tensor product of the latter multiplet given above and the coupling-function multiplet:

$$\Sigma_0\otimes(A^{ijkl};\chi^{ijkl};F^{ijkl}) = (A_0A^{ijkl};A_0\chi^{ijkl}+\chi_0A^{ijkl};A_0F^{ijkl}+F_0A^{ijkl}-\bar{\chi}_0\chi^{ijkl})$$

The complete Q-supersymmetric extension of L_4 requires adding terms which result as permutations of $ijkl \leftrightarrow ilkj \leftrightarrow kjil \leftrightarrow klij$ keeping $\eta_{ij}\eta_{kl}$ fixed.

We have not finish yet. One might ask the natural question : What does the above quartic-action has to do with Dolan and Tchrakian's action? More precisely, how do we interpret and/or dispense of the extra fields which comprised the "coupling"function? To answer this question properly we must first ask ourselves what are the requirements to have a spinning membrane action.(Q-spinning in our case). These are:

1).Linearly realized supersymmetry(Q-supersymmetry). 2). Absence of R terms. 3).Polynomial in the fields. 4).Eliminating the auxiliary fields, $\frac{\partial L}{\partial F^i}=0$, and setting the Fermi fields to zero we must recover Dolan and Tchrakian's action. Furthermore, the order in which we perform this should yield identical results: set Fermi fields to zero and eliminate auxiliary fields or viceversa. This is the content of subsection **4.2** where it is shown that, in fact, these four requirements are satisfied.

To conclude we have Q-supersymmetrized the Weyl-covariantized Dolan and Tchrakian's action. The kinetic terms and quartic terms are Q-invariant by construction. The latter ones were Q-invariant with the aid of an extra multiplet, the "coupling"function multiplet whose weight is precisely equal to -3 to ensure that our action is dimensionless and scale invariant. After eliminating the auxiliary fields, setting the Fermi fields to zero, and fixing the dilational gauge invariance, we retrieve the Dolan and Tchrakian Lagrangian. The main point of this paper is to show that one can only have a Q-spinning membrane, solely, if we wish to satisfy all of the requirements listed before. "$Q+S$" invariance can only be implemented in non-polynomial actions as Rocek and Lindstrom showed [5]. After subsection **4.2** we turn to the discussion concerning the presence of the b_μ terms which is crucial since now we do not have at our disposal the possibility of fixing the K-invariance to set b_μ =0. We have decided to include this discussion in the following Appendix because the whole essence of section IV is based on Q-invariance.

4.2 Determination of F^i, A_o, χ_o, F_o and b_μ.

After eliminating the auxiliary fields via their equations of motion and setting the fermions to zero we must recover our initial Weyl-covariantized Dolan-Tchrakian action (WCDT). Furthermore, the order in which perform this must yield identical results. Setting the fermions to zero, first, and eliminating the F^i field, per example, yields $F^i=0$ since we don't wish to generate constraints amongst the matter fields. The $F_oA^{ijkl};\chi_o\chi^{ijkl}$ terms vanish in this limit and we are left with the bosonic pieces in A_oF^{ijkl}. Similar conclusions hold for the quadratic terms as well. Lets vary the F^i,F_o fields and check that after the fermions are set to zero we recover the WCDT action. We assume always that the target spacetime indices are contracted with $\eta_{ij}\eta_{kl}$ and their permutations are included. The variations with respect $F_o;F^i$ are :

$$A^{ijkl}=\bar\chi^{ij}\chi^{kl}=0 \qquad (4-17)$$

$$1/2A_o\bar\psi_\mu\gamma^\mu(\partial\chi^{ijkl}/F^i)+A_o\partial F^{ijkl}/\partial F^i-$$
$$\bar\chi_o\partial\chi^{ijkl}/\partial F^i+\delta L_2/\delta F^i=0 \qquad (4-18)$$

In the last equation we used $\chi^{ij}=0$ which is a solution of (4-17) yielding F^i in terms of the matter fields, A^i,χ^i from eq-(4-11). After setting the fermions to zero, the $F^i=0$ and we recover the WCDT action. Notice that plugging the value for F^i obtained from (4-17) in eq-(4-18) yields a constraint equation amongst A_o,χ_o and the matter fields, but in no way whatsoever, we are constraining the latter fields!

Having the WCDT action still doesn't provide us with the original DT action. In the Appendix we show that if the equations of motion for the membrane coordinates obtained from the WCDT action are, indeed, the Weyl-covariantized form of the equations of motion of the DT action, then $D_\mu A_o=0$ follows immediately. Therefore, b_μ is zero in the $A_o=$ *constant* gauge. Also we show that if b_μ is varied the bosonic action is constrained to zero. Another way to see why the b_μ field is not determined via its equation of motion but from the equations which follow from the A_o,χ_o variation is the following.

Since supersymmetry rotates field equations into field equations for the members of a given supermultiplet, the A_o, χ_o variations are encoded already in the F_o variation and, therefore, cannot and should not be ignored. A variation with respect to A_o yields, after using eq-(4-17) with $\chi^{ij} = 0$ as a solution, in the expression for F^{ijkl} :

$$F^{ij}F^{kl} + b_\mu(terms) - 2g^{\mu\nu}\partial_\mu(\bar{\chi}^i\chi^j)\partial_\nu(\bar{\chi}^k\chi^l) = 0. \qquad (4-19)$$

A variation with respect to χ_o yields :

$$1/2\psi_\mu\gamma^\mu A^{ijkl} - \chi^{ijkl} = 0. \qquad (4-20)$$

It isn't difficult to verify that $\chi^{ij} = 0$ is a solution of (4-20) in consistency with (4-17) by a simple inspection of eq-(4-15) and after using eq-(4-17).

We can now see how the b_μ field is determined by eq-(4-19). Evenfurther, we can also see, once more, why we couldn't supersymmetrize, directly, the DT action. If we fix the dilational gauge invariance, $A_o = constant$, while adding compensating gauge transformations to the Q supersymmetry transformation laws, and if we use the embedding condition :$D_\mu A_o = 0$ in equations (4-17;4-18;4-19;4-20) we find that equation (4-19) is going to constrain, again, the matter fields in the $b_\mu = 0$ gauge, after one substitutes the value for F^i obtained from eq-(4-17). Therefore, we must have dilational gauge invariance in our spinning membrane !. We can still plug-in the expression $b_\mu \sim \partial_\mu ln\ A_o$ into equations (4-18;4-19) yielding A_o, χ_o in terms of the matter fields.

We have seen how the fields $F^i; A_o, \chi_o; b_\mu$ are tightly constrained through the use of the above equations. The F_o field is undetermined however, after the fermions are set to zero, the F_oA^{ijkl} vanishes in any case.

Therefore, the elimination of the auxiliary fields constrains the full quartic supersymmetric piece to vanish on shell without constraining, in any way whatsoever, the physical fields as we have shown. Despite having only a quadratic piece in our spinning membrane we cannot forget the presence of the quartic terms which appears when the F^i field is solved via equation (4-17) and when the b_μ field must obey eq-(4-19) as well. The fact that L_4 vanishes on shell might be topological in origin. These zero actions are important in Topological Quantum Field theories. This should be investigated further. Of course, we cannot say that the quartic terms of the WCDT action, after setting the fermions to zero in eq-(4-19), have to vanish. This is because eq-(4-19) was due to supersymmetry which is broken after the fermions are set to zero! We must remember that $A_o; \chi_o$ were varied because they were part of the supermultiplet which contained F_o and supersymmetry forced their variation. i.e. one cannot, simultaneously, set the fermions to zero in equation (4-19) and still use equation (4-19) because such equation ceases to be valid as soon as supersymmetry is broken (as soon as we set the fermions to zero).

Another way of rephrasing this is by saying that the A_o field was used, in the first place, to Weyl covariantize the quartic terms of the DT action and, for this reason, A_o is eliminated by fixing the dilational gauge invariance (A_o is gauged to a constant) in the WCDT action and not via its variation. Clearly, varying A_o in the WCDT action constrains the quartic terms to vanish and one has no longer a Weyl covariant extension of the DT action which was, in the first place, the reason why we introduced A_o!!! Similar conclusions hold if we add kinetic pieces $A_1(D_\mu A_0)^2$ to the WCDT action and one eliminates both the A_0 and A_1 fields; one will constrain the quartic derivative terms to vanish.

To summarize what we just said in the prevoius paragraphs : (i). After eliminating the F^i, F_o auxiliary fields from the action and substituting their values in the Q supersymmetry transformations laws yields a Q invariant action, iff, equations (4-19;4-20) are satisfied. (ii). The field A_o can be gauged to a constant but one cannot ignore the constraint equation which arises from its variation in the full Q supersymmetric action. Such constraint owes its existence to supersymmetry and, therefore, one cannot naively set the fermions to zero and still use equation (4-19)to falsely claim that the quartic terms of the WCDT action are zero !.What is zero is the full equation (4-19)- which follows from supersymmetry- and it is no longer valid as soon as the fermions are set to zero.

We have shown that, in fact, eliminating the auxiliary fields via their equations of motion and setting the fermions to zero yields the WCDT action. The spinning membrane effectively consists of a Weyl covariant quadratic piece like it occurs in the spinning string; however the background field b_μ and the scalar coupling, A_o are tightly constrained by a set of equations which had their origins in the quartic terms of the Q supersymmetric WCDT action. These quartic supersymmetric terms are zero on shell. We couldn't fix, first, the dilational gauge invariance in the supersymmetric action since constrains will reappear amongst the matter fields. For this reason one must have dilational gauge invariance in the spinning membrane.

Having only Q supersymmetry isn't as bad as it seems. In Poincare supegravity one does not have Q and S supersymmetry invariance, separately, but it is only a combination of Q and S which is preserved, the so called $Q+S$ sum rule. Since conformal invariance was crucial in our construction it is warranted to study the quantum case and see how conformal anomalies will yield information about the critical dimension; presummably this might single out eleven dimensions.

5 APPENDIX

The discussion in section IV cannot be complete unless we study in detail the behaviour of our final action due to the presence of the b_μ terms and derive from first principles the embedding condition :$D^c_\mu A_o = 0$. To begin with, we have two cases to consider:

1-. The case where b_μ decouples from the action and from the Q-supersymmetry transformation laws of the action. An example of this is the action given by eq-(4-9) for the particular case that one chooses the multiplet

$$\Sigma \otimes T(\Sigma)$$

with a net weight equal to $\lambda = 2$. One can see, explicitly, by simple inspection of eqs-(4-1;4-2;4-3;4-4) and all of the eqs. (4-5a-e) that the b_μ terms decouple. Therefore, there is no explicit b_μ dependence in the action (4-9) and, thus, we have implemented K-invariance. Furthermore, we can

see that under Q-supersymmetry , eq.(4-9) contains the terms: δF yields a term $\bar{\epsilon}\gamma^\mu(D_\mu - (2+\frac{1}{2})b_\mu)\chi$ whose b_μ term is

$$-(2+\frac{1}{2})\bar{\epsilon}\gamma^\mu b_\mu \chi.$$

A factor of $-b_\mu$ cancels against the b_μ terms contained in the $\omega(e;\psi;b_\mu)$ leaving us with a net factor

of $-\frac{3}{2}$. Whereas the second term of (4-9) yields , upon variation of the gravitino using (4-7), the following term:

$$\frac{1}{2}\bar{\epsilon}\gamma^\mu b_\mu \chi.$$

plus another factor of $+b_\mu$ stemming from the spin-connection leaving a net factor of $\frac{3}{2}$. It is clear that b_μ does also decouple from the transformation laws. However, if these didn't , we still have K-invariance which allows to set b_μ=0 and everything is fine.

2-. The case when b_μ does not decouple from the action but it does from the transformation laws to ensure Q-invariance (since we can no longer choose the gauge $b_\mu = 0$; these b_μ terms must cancel out since Q-invariance was not broken explicitly). This is our case. We bring to the attention of the reader that $\Lambda = T(\Sigma \otimes \Sigma)$ is not K-inert. The simplest way to see this is by looking at the superconformal algebra in three dimensions. The Jacobi identity implies :

$$[\Lambda, [Q, K]] + [Q, [K, \Lambda]] + [K, [\Lambda, Q]] = 0.$$

Since Λ is Q invariant $\Rightarrow$:

$$[\Lambda, Q] = 0$$

Because

$$[Q, K] \sim S. \quad [\Lambda, S] \neq 0$$

we have

$$[K, \Lambda] \neq 0.$$

Therefore we have b_μ terms in our final expressions for the action. The question is: How bad is this? A careful study shows that the presence of the b_μ terms is not harmful at all. The reasoning goes as follows: After the elimination of the F^i auxiliary fields, per example, and setting the fermions to zero, we have terms of the form :

$$(\partial_\mu A^i - \lambda b_\mu A^i)^2 - A_o(\partial_\mu A^i - \lambda b_\mu A^i)^4.$$

If one attempts to eliminate the b_μ through its variation one would arrive at a zero action. Eliminating b_μ from the above equation, after factoring A^i, yields an expression of the form :

$$(D_\mu A^i) - 2A_o(D_\mu A^i)^3 ..... = 0$$

A particular non trivial solution is

$$D_\mu A^i = 0$$

for all values of $i = 1, 2, 3........d$. This implies that :

$$b_\mu \sim \partial_\mu \ lnA^i$$

for all values of $i = 1, 2......d$. Therefore the A^i are constrained to satisfy the condition: $A^j = k_j A^1$ for $j = 2, 3....d$ constants k_j. Since we still have at our disposal the freedom to fix the dilational gauge invariance, we can set $A^1 = constant$ forcing all of the rest A^j to be constant and, hence, the action would be constrained to vanish. This is unaccetable. Analogous results would have been obtained upon the elimination of b_μ from the Weyl covariant form of the DNG action as well.

Therefore, in view of the above arguments, the correct procedure to follow is to fix the local scale invariance by setting:

$$-A_0(x) = 1.$$

and, simultaneously, set b_μ to zero. i.e; one chooses to have trivial background-gauge field configurations (pure gauge ones) without dynamical degrees of freedom. This presupposes the fact that one can find a gauge where (simultaneously) the conformal compensator can be gauged to a constant and the b_μ field to zero. This can in fact be achieved by choosing for gauge parameter the quantity:

$$\Lambda = \frac{1}{3} ln(-A_0)$$

It is straightforward to verify that b_μ can be gauged to zero and A_0 to -1 simultaneously. Both of these conditions can be condensed into a single equation :

$$D_\mu^{Weyl}(-A_0) = \partial_\mu(-A_0) + 3b_\mu(-A_0) = 0$$

since the weight of A_0 is -3.

As we promised earlier we are going to derive the constraint on A_o from first principles, from an action. Our guiding principle will be the on-shell dilational gauge invariance of the WCDT equations of motion : the former must be the Weyl covariant extension of the DT equations of motion. In particular, the contribution to the equations of motion stemming from the quartic derivative terms of the WCDT action are of the form :

$$D_\mu(\delta S_4/\delta(D_\mu A)) \sim (D_\mu A_o)(D_\mu A)^3 + A_o D_\mu(D_\mu A)^3.$$

The above expression is Weyl covariant as it should be. We have assumed that there are no boundary terms in our action and that the fields vanish fast enough at infinity, etc.... As it is usual in these variational problems we have integrated by parts and generalized Stokes law to the Weyl covariant case.

Similarly, the coresponding terms associated with the DT action are :

$$\partial_\mu(\delta S_4/\delta(\partial_\mu A)) \sim \partial_\mu(\partial_\mu A)^3.$$

If one set of equations are, indeed, the Weyl covariant extension of the other set, then $D_\mu A_o = 0$ follows immediately. Therefore, the DT action admits a Q supersymmetric extension, iff, the fields, A_o, b_μ satisfy the embedding condition : $D_\mu A_o = 0$. This completes the results of subsection **4.2**.

To finalize this Appendix we point out that the only obstruction in setting b_μ to zero must be topological in origin. We saw in section III that it was the elimination of S which originated the constraint $\bar{\chi}\chi$ =0. Such S term had the same form as a fermion-condensate. Whereas here, upon the "trade-off" $b_\mu \rightarrow \frac{1}{4}\gamma_\mu S$, we may encounter topological obstructions in setting b_μ =0 gobally and, henceforth, in Q-supersymmetrizing the Dolan-Tchrakian action. i.e; to obtain the exact bosonic limit from the Q-supersymmetric action.

It is warranted to study the topological behaviour of these 3-dim gauge fields and see what connections these may have with Topological Massive Gravity [7], Chern-Simmons 3-dim Gravity and with other non-perturbative phenomena in three dimensions.

6 ACKNOWLEGEMENTS

We thank Yuval Ne'eman and Luis J. Boya for helpful discussions throughout the course of this work.

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